

6. If $x(t)$ represents the position of a particle along the x -axis at any time, t , fill in the blanks in the statements below with the best answer so that they become true facts (not opinions).

(a) "Initially" means when (time) $t = 0$.

(b) "At the origin" means (position) $x(t) = 0$.

(c) "At rest" means $x'(t) = 0$ $= 0$.

(d) If the velocity of the particle is positive, then the particle is moving to the right.

(e) If the velocity of the particle is negative, then the particle is moving to the left.

(f) To find average velocity over a time interval, divide the change in position by the change in time.

(g) Instantaneous velocity is the velocity at a single moment (instant) in time.

(h) If the acceleration of the particle is positive, then the velocity is increasing.

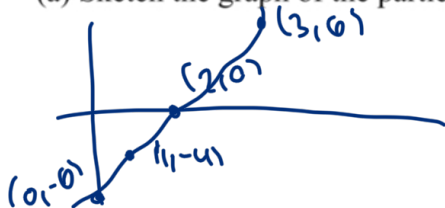
(i) If the acceleration of the particle is negative, then the velocity is decreasing.

(j) In order for a particle to change directions, the velocity must change signs.

(k) One way to determine total distance traveled over a time interval, when given the position function or graph, is to find the sum of the absolute values of the differences in position between all resting points.

7. If the position of a particle along a horizontal line is given by $x(t) = t^2 + t - 6$ for $0 \leq t \leq 3$

(a) Sketch the graph of the particle's position on the given interval.



(b) What is the particle's displacement on the given interval? Show the work that leads to your answer.

$$x(3) - x(0)$$

$$12$$

(c) Find the total distance traveled by the particle on the given interval. Show the work that leads to your answer.

$$v(t) = x'(t)$$

$$v(t) = 2t + 1$$

$$\text{let } v(t) = 0$$

$$0 = 2t + 1$$

$$t = -\frac{1}{2}$$

8. The data in the table below gives selected values for the velocity, in meters/minute, of a particle moving along the x -axis. The velocity v is a differentiable function of time t .

velocity is differentiable so velocity is continuous

Time t (min)	0	2	5	6	8	12
Velocity $v(t)$ (meters/min)	-3	2	3	5	7	5

- (a) At $t = 0$, is the particle moving to the right or to the left? Justify.

At $t=0$, the particle is moving left
 $v(0) < 0$

- (b) Is there a time during the time interval $0 \leq t \leq 12$ minutes when the particle is at rest? Explain your answer.

$v(t)$ is continuous.
 $v(0) < 0$ and $v(2) > 0$ so $v(t) = 0$ using the Intermediate Value theorem. So the particle must be at rest at that moment.

- (c) Use data from the table to find an approximation for $v'(10)$ and explain the meaning of $v'(10)$ in terms of the motion of the particle. Show the computations that lead to your answer, and indicate units of measure.

$$v'(10) \approx \frac{v(12) - v(8)}{12 - 8}$$

$$v'(10) \approx -\frac{1}{2} \frac{\text{m/min}}{\text{min}} = -\frac{1}{2} \text{ m/min}^2$$

$$v'(10) = a(10)$$

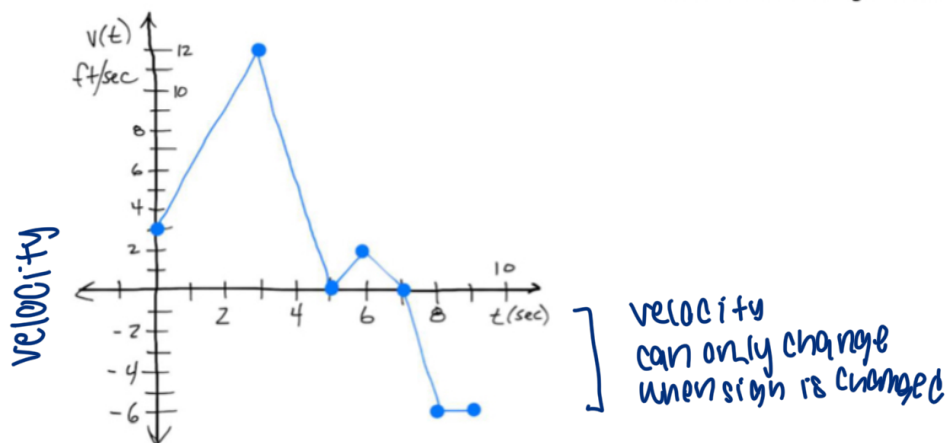
$-\frac{1}{2} \frac{\text{m/min}}{\text{min}}$ means that on average the particle loses $\frac{1}{2} \text{ m/min}$ each minute from $t=8$ to $t=12$.

- (d) Find the average acceleration of the particle for $8 \leq t \leq 12$ min. Explain what this number means in terms of the particle's velocity on that interval.

$$\frac{v(12) - v(8)}{12 - 8}$$

- (e) Let $a(t)$ denote the acceleration of the particle at time t , such that $v'(t) = a(t)$. Is there guaranteed to be a time $t = c$ in the interval $0 \leq t \leq 12$ such that $a(c) = 0$? Justify your answer.

I do not know the Mean Value Theorem.



9. The graph above represents the velocity v , in feet per second, of a particle moving along the x -axis over the time interval for $0 \leq t \leq 9$ seconds.

(a) At $t = 4$ seconds, is the particle moving to the right or left? Justify.

At $t=4$ the particle is moving to the right

(b) At what time(s) is the particle at rest? Justify.

$V(5)=0$ and $V(7)=0$ so the particle is at rest at $t=5$ and $t=7$.

(c) At what time(s) does the particle change direction? Justify.

The particle changes direction at $t=7$ because the velocity changes from positive to negative

(d) On what open intervals $0 < t < 9$ is the particle moving left? Justify.

When velocity is negative

(e) What is the acceleration of the particle at $t = 4$ seconds? Show the work that leads to your answer.

$$a = v'(t) = \frac{v(5) - v(3)}{5 - 3} = \frac{0 - 12}{2} = -6 \frac{\text{ft/sec}}{\text{sec}}$$

(f) On what open intervals $0 < t < 9$ is the acceleration of the particle positive? Justify.

on $(0, 3)$ and on $(5, 6)$ acceleration is positive because on those intervals $v(t)$ is increasing.

(g) What is the average acceleration of the particle over the interval $t \in [3, 6]$ seconds? Show the computations that lead to your answer, and indicate units of measure.

$$\frac{v(6) - v(3)}{6 - 3} = \frac{2 - 12}{6 - 3} = -\frac{10}{3}$$

(h) On what open intervals $0 < t < 9$ is the **speed** of the particle decreasing? Justify.

(i) Without knowing the initial position of the particle, is it still possible to determine the time at which the particle is farthest right for $0 \leq t \leq 9$? If not, explain. If so, find this value of t , and explain.